

Temperature Fields in Extruder Dies With Circular, Annular, or Slit Cross-Section

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Viscous dissipation causes significant temperature increases in polymers while they are flowing through extruder dies. The development of the temperature field has been studied numerically using the dimensionless parameters Na and Gz . With the annular geometry (parameter: ratio of radii κ) the well known developing temperature fields in capillaries could be compared with the developing temperature fields in an annulus and in a plane slit. The shear dependence of the viscosity is described by a "power law" and the temperature dependence by an exponential function. A simple graphical method is suggested for estimating temperature fields in extruder dies, making use of the fully-developed temperature field for very long dies as a reference state. For the demonstration of results, a power law exponent $n = 0.4$ has been chosen.

INTRODUCTION

In extrusion, a molten polymer is forced through a die by a pressure gradient. The specific energy needed to maintain the flow is equal to the pressure drop along the die. Most of this energy is being dissipated, mainly in the zones of large shear stress close to the walls. For the design of extruder dies, one wants to know:

- How much the temperature field in the melt is influenced by the dissipation of flow energy and by heat conduction towards the walls, and

- How one can change the temperature field through geometry and through the thermal boundary conditions.

For the flow in dies used in pipe extrusion, monofilament extrusion, sheet extrusion, film blowing, blow molding, capillary rheometry, etc., (i.e. flow in channels of annular, circular, or slit cross-section) we can predict the temperature fields quite accurately. Knowing the developing temperature field, it is possible to calculate the temperature effects on the velocity field, on the shear stresses, and on the pressure drop.

The developing temperature field in a capillary has been studied analytically by a number of authors: In a first solution Brinkman (1) assumed the viscosity and the thermal properties to be constant. Bird (2) extended the analysis to generalized Newtonian fluids, where the shear viscosity depends on the shear rate by a "power law." Developing tem-

peratures have been calculated for isothermal and for adiabatic capillary walls. Toor (3) studied the influence of cooling resulting from assumed spontaneous expansion during flow. Density and expansion coefficient were taken from equilibrium thermodynamics data at the pressure p in molten polymers at rest. Gee and Lyon (4) used a numerical method to take into account temperature and shear dependence of viscosity, zero-shear-rate viscosity (in a modified power law), and cooling from expansion. For the momentum and the mass balance, the fluid has been assumed to be incompressible. The wall temperature has been chosen to be constant, but differing from the fluid temperature at the inlet. Recently Galili and Takserman-Krozer (5) used a perturbation method to get a first order approximation of the velocity and pressure distribution for a Newtonian fluid with temperature dependent viscosity, constant density and constant thermal properties.

Numerical methods seem to be the most adequate for calculating the developing temperature field. Due to the difficulties in applying knowledge from published numerical solutions to their individual problems, several authors (6-12) developed their own numerical capillary program using mostly explicit difference methods. This study uses an iterative implicit method which conserves mass, energy, and momentum.

Experimental studies have been made on the effects of temperature on the pressure drop (1, 4, 6, 8, 11, 13), the average temperature increase (9, 11, 13), the wall temperatures (4, 6, 13) the wall heat flux (8, 14), and the radial temperature distribution

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(6, 8, 15-17). The experiments seem to support the corresponding analytical studies quite well.

Under certain experimental conditions it is possible to have an adiabatic wall (8), but it doesn't seem to be possible to maintain isothermal wall conditions when viscous dissipation is significant (6, 11, 13, 15, 18). In applications, the wall condition is in between adiabatic and isothermal; the most needed information from experiments is about the actual thermal boundary conditions, which have to be used in a numerical program.

In contrast to the rather complete investigation for the capillary, the developing temperature field in an annulus and in a plane slit has seen only limited study. Seifert (10) calculated the developing temperature field for a Prandtl-Eyring-fluid flowing in a plane slit; his main interest has been on the heat flux at the walls for symmetric and asymmetric wall temperatures. Cox and Macosco (11) investigated the temperature effects in dies of this geometry. They take into account shear and temperature dependence of viscosity as well as density changes. However, they describe their results by means of the heat transfer coefficient, which has no physical meaning for flow with viscous dissipation. Due to the large viscous dissipation in the layers next to the wall, there exists no simple relation between temperature gradient at the wall and difference between wall temperature and bulk temperature.

The present investigation wants to show the common temperature effects in the flow of molten polymers in different axisymmetrical die geometries. The numerical solution of the problem is presented graphically. Thus, by graphical means, it is possible to predict the increase of the average temperature in the die as well as the temperature field, using the values of the dimensionless parameters shown below. Recently, the numerical program has been applied to the design of dies for a capillary viscosimeter (18).

ASSUMPTIONS, SYSTEM OF EQUATIONS

A large number of dies are described geometrically by an annulus with the ratio of the radii

$$\kappa = r_i/r_a \quad 0 \leq \kappa < 1 \quad (1)$$

if one includes the limiting cases $\kappa = 0$ (die with circular cross-section) and $\kappa \approx 1$ (slit die). The cylindrical coordinate system is the natural one for the problem (Fig. 1). The position in the die is given by the dimensionless coordinates

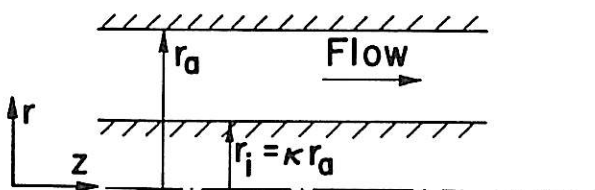


Fig. 1. Annular flow channel.

$$\text{radially:} \quad Y = \frac{r - r_i}{r_a - r_i} \quad 0 \leq Y \leq 1 \quad (2)$$

$$\text{axially:} \quad Z = \frac{z}{l \cdot Gz} \quad 0 \leq Z \leq Gz^{-1} \quad (3)$$

The dimensionless coordinate Y is very powerful for describing the influence of the geometry, i.e., the influence of κ : the isothermal velocity field (Fig. 2) in a plane slit ($\kappa = 0.99$) is symmetrical to the center of the slit ($Y = 0.5$). With decreasing κ the maximum of the velocity field moves towards the axis of the annulus. For the die with circular cross section ($\kappa = 0$) the velocity field is symmetrical to $Y = 0$.

The value of Z indicates, as it will be shown in the following, how much the temperature field is developed along the die. The shear dependence of the viscosity* is described by a "power law" and the temperature dependence by an exponential function

$$\eta = \bar{\eta} \exp[-b \cdot (T - T_o)] \left(\frac{h}{\bar{v}_z} \frac{\partial v_z}{\partial r} \right)^{n-1} \quad (4)$$

where

$$\bar{\eta} = \eta \left(\frac{\bar{v}_z}{h}, T_o \right)$$

is the reference viscosity.

Instead of this, it also would be possible to use a viscosity table for the computer program. Also the temperature dependence of n can be included, if one uses the computer program instead of the graphical method proposed. Throughout this paper, the power law exponent is taken to be $n = 0.4$ (which

* Viscosity is the one of three material functions describing the shear stress in "steady shear flow" (defined by a history of constant temperature and constant shear rate). The flow in dies is nearly steady shear flow only, since (a) the temperatures change along the stream lines, and (b) the shear rates along stream lines change slightly due to the temperature changes. The stationary layer of polymer next to the wall, however, is subjected exactly to steady shear flow, because temperatures and shear rates are constant with time ($\partial/\partial t = 0$).

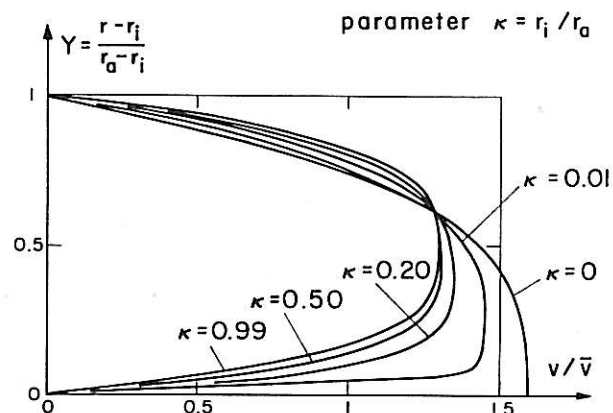


Fig. 2. Velocity profile of an isothermal power law fluid in an annulus; $n = 0.4$.

is, for instance, a reasonable value for low density PE at 190°C).

The equations of change describe the flow in dies. They were simplified by the following assumptions

- Incompressible, isotropic fluid with constant thermal properties. (The temperature and the pressure dependence of the density and of the thermal properties can be accounted for in the numerical program as long as material data is available.)

- Steady laminar flow ($\partial/\partial t = 0$).
- Rotational symmetry ($\partial/\partial\theta = 0$).
- Radial velocity component $v_r \ll v_z$; $\partial v_r/\partial r$, $\partial v_r/\partial z$, $\partial v_z/\partial z \ll \partial v_z/\partial r$.
- Kinematically developed velocity profile in the entrance section. (For low Reynolds number flow of inelastic fluids, this has been shown (19) to be a reasonable assumption.)
- The pressure is a function of z only.
- No slip at the walls.
- Heat is transported axially by convection and radially by conduction only.

The system of partial differential equations for the problem is the following (after applying the assumptions),

Equation of continuity:

$$0 = \frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} \quad (5)$$

Equation of motion:

$$0 = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial r}\left(r\eta\frac{\partial v_z}{\partial r}\right) \quad (6)$$

Equation of energy:

$$\frac{v_z}{a} \frac{\partial T}{\partial z} = \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\eta}{k}\left(\frac{\partial v_z}{\partial r}\right)^2 \quad (7)$$

The average velocity is

$$\bar{v}_z = \frac{2}{r_a^2 - r_i^2} \int_{r_i}^{r_a} rv_z dr \quad (8)$$

The boundary and the initial conditions for the velocity and for the temperature field are:

Boundary conditions:

$$\left. \begin{aligned} v_z(r_i, z) = v_z(r_a, z) = 0 \\ v_r(r_i, z) = v_r(r_a, z) = 0 \\ T(r_i, z) = T_o \text{ or } \partial T/\partial r|_{r_i, z} = 0 \\ T(r_a, z) = T_o \end{aligned} \right\} 0 \leq z \leq l \quad (9)$$

Initial conditions:

$$\left. \begin{aligned} v_z(r, 0) = v_{z,0}(r) \\ T(r, 0) = T_o \end{aligned} \right\} r_i \leq r \leq r_a$$

The equation of motion (Eq 6) and the equation of energy (Eq 7) are coupled through the temperature dependent viscosity. The extent of the coupling increases with the value of the Nahme number (20)

$$Na = \frac{\bar{v}_z^2 b \bar{\eta}}{k} \quad (10)$$

which compares the dissipation term and the conduction term in Eq 7. For values of Na greater than 0.5 to 1 (depending on geometry and thermal boundary conditions) the viscous dissipation leads to significant viscosity changes. For smaller values of Na isothermal conditions can be actually achieved in extruder dies by taking a constant inlet temperature equal to the temperature of the die wall. In this case the equation of motion can be integrated easily, even for the annular geometry (21).

In some studies the Brinkman number (1) $Br = \bar{v}_z^2 \eta / kT_o$ has been used instead of the Nahme number. But Br contains the arbitrary temperature level T_o and may therefore have very different values for similar processes. The value of Br does not give information on the extent of the coupling between the equation of motion and the energy equation. (Note that the Nahme number sometimes is called the Griffith number after Griffith (22), who used the same dimensionless group in one of the later applications.)

The energy equation (Eq 7) contains a convection, a conduction, and a dissipation term. By comparing the convection and the conduction term one gets the Grätz number (23)

$$Gz = \frac{\bar{v}_z h^2}{al} \quad (11)$$

which has been included in the dimensionless form of the z -coordinate. A large value of Gz means that heat convection in the flow direction is more important than conduction towards the walls. $Gz = 100$ is a common value for extruder dies. (Note that some authors define the Graetz number $Gz \cdot \pi$.)

In very short dies ($Gz > 10^3$) conduction of heat becomes negligible compared with convection, i.e., the conduction term in Eq 7 can be neglected and the ratio Na/Gz becomes the dimensionless parameter. If the pressure drop Δp in the die is known from experiments, one can estimate the importance of viscous dissipation from the dimensionless product $\Delta p \cdot b / \rho c$ (which is equivalent to Na/Gz). For values $\Delta p \cdot b / \rho c > 0.5$ the viscous dissipation leads to significant viscosity changes.

The boundary and the initial conditions for the temperature field are not known in general. If there are temperature data for the entrance and along the walls, they can be used in the numerical program. But most often one has to guess these conditions to enable an estimate of the temperature profiles; for this presentation I took two idealized boundary conditions: constant wall temperature and zero temperature gradient at the inner wall, respectively. The outer wall has been assumed to be isothermal. At the entrance, a constant temperature equal to the wall temperature is assumed, which can be achieved in extruders through a mixing element at the tip of the extruder screw.

One gets a system of three nonlinear partial differential equations and an integral. It has been solved numerically by an iterative implicit differ-

ence method. The solution procedure and the experimental check of the solution has been described in a previous report (24) on helical flow of molten polymers. The temperature effects in helical flow and in die flow are very much the same, and it doesn't seem to be necessary to perform additional experiments for the different die geometries.

RESULTS

The temperature field in dies is determined (1) by convection in axial direction, (2) by conduction in radial direction, and (3) by viscous dissipation. In Figs. 3-5, examples for developing temperature fields are shown for two values of the Nahme number. The temperature of the polymer melt increases in the flow direction by the effect of viscous dissipation. The radial temperature gradients become increasingly steeper up to the point where the fully developed temperature field ($\partial T/\partial z = 0$) is reached. From that point on, all the dissipation is balanced by radial conduction; convection has no effect, because in the direction of flow the temperature doesn't change any more.

Figure 6 shows the increase of the average temperature along the die, normalized by the maximum possible temperature increase. For flow in a slit die, the highest and lowest curve describe the development at different thermal boundary conditions. Both curves have about the same shape, but in a plane slit with two isothermal walls the fully developed temperature field is reached in about one fourth of the length compared with a slit with one wall adiabatic and one wall isothermal.

For smaller values of κ , the corresponding curves for the two kinds of boundary conditions move together, which can be explained easily: the thermal boundary condition at the inner wall of the annulus becomes less and less important, when the inner surface decreases with smaller values of κ . The curve in the middle shows the average temperature increase in a pipe with circular cross section ($\kappa = 0$).

Surprisingly, the development of the (dimensionless) average temperature depends very little on the values of Na , i.e., the curves for $Na = 1$ shown in Fig. 6 are practically identical with curves for $Na = 10$. The shape of the radial temperature pro-

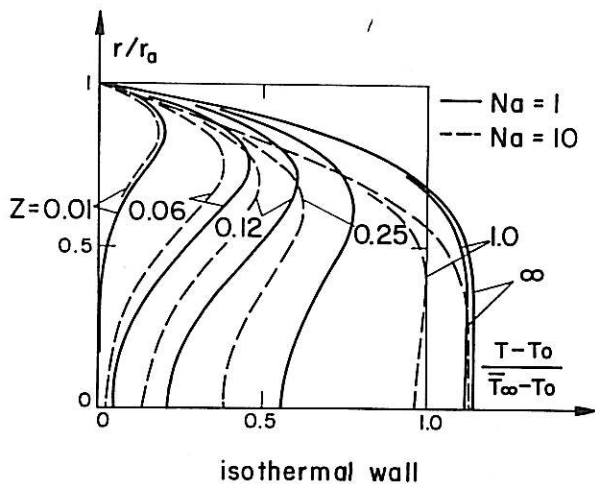


Fig. 3. Developing temperature field in a pipe; $n = 0.4$.

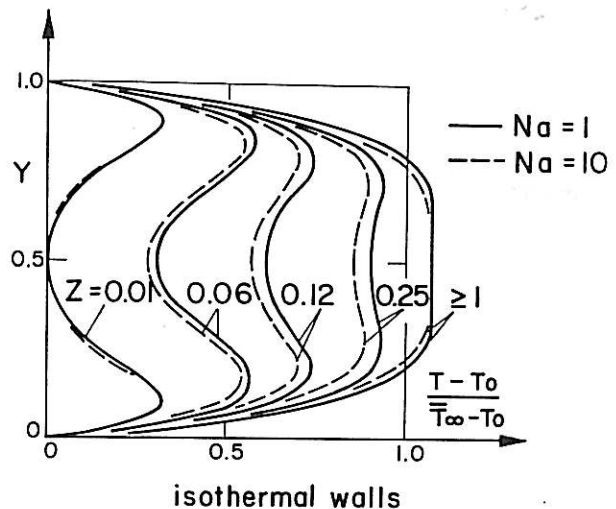


Fig. 5. Developing temperature field in a plane slit; $n = 0.4$.

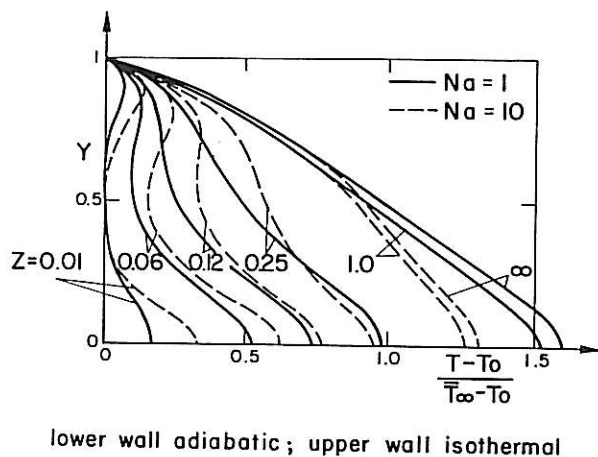


Fig. 4. Developing temperature field in a plane slit; $n = 0.4$.

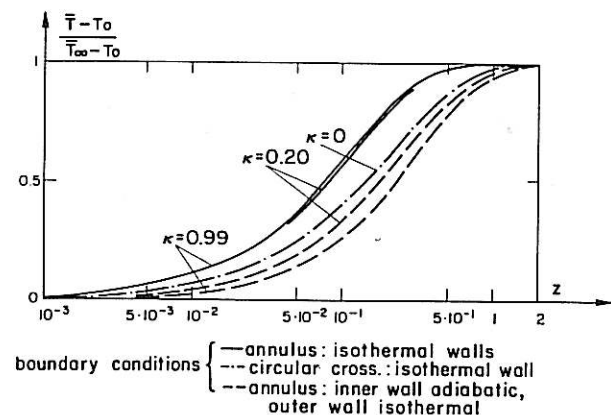


Fig. 6. Average temperature increase along channels with different geometries and different thermal boundary conditions; $Na = 1$; $n = 0.4$.

files (Figs. 3 to 5), however, depends on Na. The dimensionless temperature rises are all in the same order of magnitude due to the choice of $(\bar{T}_\infty - T_o)$ as a normalizing factor; the absolute values of the temperature rise $(T - T_o)$ increase very much with increasing Na.

In general, extrusion dies are too short ($Z < 1$) to obtain full thermal development. Therefore, special attention has been paid to the initial development of the temperature field. The fully developed temperature field is used as a reference state only. A survey of the fully developed temperature field is given in the literature (25).

Figure 7 shows the maximum possible temperature increase $b \cdot (\bar{T}_\infty - T_o)$ as a function of Na, κ , and the thermal boundary conditions. Depending on the thermal boundary conditions, the plain slit may give the highest or the lowest temperature increase. For decreasing values of κ the curves depend less and less on the boundary condition at the inner wall.

GRAPHICAL METHOD

For a certain value of Na, which can be calculated from the average velocity in the die and from the properties of the molten polymer, the maximum possible temperature increase in a long die can be determined from Fig. 7. Then the value of Gz has to be calculated to give the range of Z for the die and the average temperature increase in that range (from Fig. 6). Figures 3 to 5 show examples of

graphs for the estimation of the radial temperature profiles.

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NOMENCLATURE

- a = thermal diffusivity, m²/sec
- b = temperature coefficient of viscosity, K⁻¹, $-\partial\eta/\eta\partial T$, see Eq 4
- Gz = Graetz number, $\bar{v}_z h^2 / a l$ [—]
- h = $r_a - r_i$ = gap width, m; $h = r_a$ for circular cross-section
- k = thermal conductivity, J/msK
- l = die length, m
- n = power law exponent, see Eq 4
- Na = Nahme number, $\bar{v}_z^2 b \bar{\eta} / k$ [—]
- p = pressure [N/m²]
- r, r_o, r_i = radial coordinate, outer and inner radius, m
- T = temperature, K; $\bar{T} = 2/(1 - \kappa^2) \int_{\kappa}^1 T v_z r / (\bar{v}_z r_a) d(r/r_a)$
- v_r, v_z = velocity components, m/s
- \bar{v}_z = average axial velocity, m/s
- Y = coordinate, see Eq 2
- z, Z = coordinate, see Eq 3
- $\kappa = r_i / r_a$ = ratio of radii
- η = viscosity, Ns/m², see Eq 4
- $\bar{\eta}$ = viscosity at reference shear rate, \bar{v}_z / h , and reference temperature T_o

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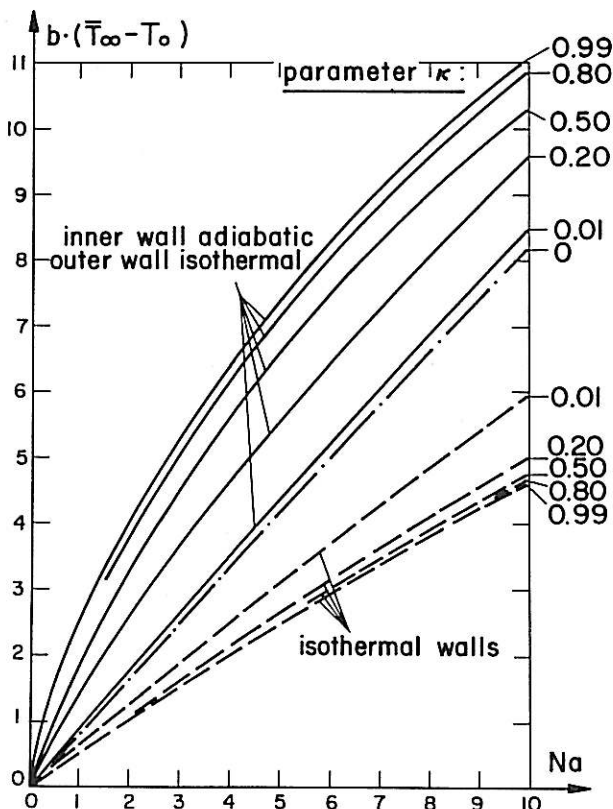


Fig. 7. Average temperature increase in long channels; n = 0.4.

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